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Little string thermodynamics

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Abstract: We argue that the holographic dual to little string theories at finite temperature suffers from a Gregory-Laflamme like instability, providing an alternative explanation to the results of hep-th/0012258.

Keywords: Field Theories in Higher Dimensions, Black Holes in String Theory.

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Contents

1. Introduction

Little string theories (LSTs) are defined to be the theory on the world-volume of NS5-branes in the limit of vanishing string coupling $g_{\rm str} \to 0$, [1] (for a review see [2]). In this limit it is argued that the world-volume degrees of freedom decouple from the bulk dynamics. The LSTs are non-local field theories [3] without gravity and one hopes to understand the physical consequences of non-locality from them as a precursor to being able to address the question in the general setting of quantum gravity. An alternate definition of LSTs can be given by looking at string theories compactified on singular Calabi-Yau spaces [4].

Despite having successfully defined decoupled non-local theories on the world-volume of NS5-branes, it has proven quite hard to extract substantial information about the dynamics of the theory. Efforts in this direction include formulation of a light-cone description [5], which has proved useful in extracting the chiral operator spectrum and the formulation of a holographic dual in terms of string theories in linear dilaton backgrounds [6]. The holographic description has been used to extract some correlation functions [7] and has been extended to more general theories with fewer supersymmetries [8, 9].

In conventional field theories one learns much about the underlying physics by subjecting matter to extreme conditions, high energies or high temperatures. One would hope that under similar treatment one could learn something about the universal features of LSTs. In particular the thermal behavior of LSTs has been addressed by various authors. It was argued in the work of [12] that the LSTs at finite temperature can be realized in terms of the CGHS black hole [10]. As a consequence of this the holographic prescription of [6] implies that the finite temperature LST is holographically dual to string theory on the CHS tube [11] capped off by a horizon (the two-dimensional cigar geometry in euclidean metric), along with a SU(2) WZW model with level prescribed by the number of NS5-branes and a free CFT for the longitudinal directions of the brane. Actually, it is not quite true that this background describes the LST at finite temperature. In fact as we shall review LST in this background is precisely dual to LST at its Hagedorn temperature [12]. The tree level thermodynamics has therefore, entropy proportional to energy, implying an exponential growth of states.[1] To get a better understanding of the system one therefore has to look at higher order corrections to thermodynamics, which has been investigated in [14]–[17]. We shall in the present work try to reinterpret some of the results presented in [16].

The punch line of [16] was that the thermal ensemble corresponding to LSTs at Hagedorn temperature exhibits an instability; it has negative specific heat. This was interpreted to signal the presence of a tachyonic mode in string theory about the background. Using the fact that the CFT in the background could be explicitly solved (the euclidean cigar background is a $\mathrm{SL}(2)/\,\mathrm{U}(1)$ coset), the authors exhibited a mode that wound once around the euclidean time direction, whose mass at tree level was shown to vanish [16, 9]. The instability at one-loop was supposed to be due to this mode becoming tachyonic (mass generation at one-loop is possible thanks to spacetime supersymmetry having been broken due to the thermal boundary conditions).

This is not a satisfactory state of affairs, since as we shall see supergravity is perfectly valid at the energy scales under question. It seems unnatural to have a geometry which is manifestly nice and smooth, wherein one would have a stringy tachyonic mode. We propose rather that the instability has its origins in supergravity itself. A concrete statement of the proposal is:

"The supergravity dual to LST at Hagedorn temperature suffers from a Gregory-Laflamme (GL) [18, 19] like instability, thereby causing the thermal ensemble to be unstable."

The GL instability in question is argued to be marginal, in the sense that it is a massless or zero frequency mode that one finds in the classical fluctuation analysis at tree level. We claim that this mode is potentially capable of acquiring a tachyonic mass at one-loop, taking responsibility for destabilizing the thermal ensemble. So we differ from the interpretation of [16] in origins of the massless mode; while the authors of [16] would like the massless mode to be coming from a stringy excitation which is localized at the tip of the euclidean cigar, our contention is that it is just a metric perturbation about the classical background.

[1] The exponential growth of states of LST was noted in the DLCQ description in [13].

We have not managed to explicitly demonstrate that the metric fluctuations about the background of interest have indeed a zero frequency mode that is capable of explaining the origins of the instability. However, we argue based on a recent paper of Reall [20]; wherein it is claimed that near-extremal NS5-branes suffer from the GL instability, that is very plausible that in the decoupling limit for finite temperature LSTSs [12], the relevant mode survives. Note that this interpretation is along the lines of the conjecture of [21, 22], arguing that thermodynamic instabilities in field theories should correspond to classical instabilities of the dual spacetime geometry.

The plan of the paper is as follows: We first begin with a review of tree-level thermodynamics of LSTs and then proceed to describe the calculations of [16] briefly. We discuss their interpretation of the result and argue that the instability would persist if one took the supergravity limit of the string theory partition function derived in [16]. Hence the origins of the instability must lie within reach of supergravity and proceed to argue that this is indeed the case aided by the supporting evidence of [20].

2. Review of LST thermodynamics

2.1 Classical thermodynamics

The metric of the non-extremal NS5-brane in the string frame is [23]

$$\begin{aligned} ds^2_{\rm str} &= -f(r)dt^2 + dy_5^2 + A(r)\left(\frac{dr^2}{f(r)} + r^2 d\Omega_3^2\right) \\ e^{2\Phi} &= g_{\rm str}^2 A(r) \\ f(r) &= \left(1 - \frac{r_0^2}{r^2}\right) \\ A(r) &= \left(1 + \frac{N l_s^2}{r^2}\right). \end{aligned} \tag{2.1}$$

The background has in addition a constant flux of the NS-NS field strength piercing the $\mathbf{S^3}$, which we shall not write explicitly. The parameter r_0 is the radius of the horizon and is related to the energy density above extremality as

$$\frac{E}{V_5} = \frac{1}{(2\pi)^5 \alpha'^3}\left(\frac{N}{g_{\rm str}^2} + \frac{r_0^2}{g_{\rm str}^2 \alpha'}\right). \tag{2.2}$$

The decoupling limit [12] is defined by scaling the asymptotic value of the string coupling and the horizon radius to zero, while holding the the energy density above extremality fixed in string units, i.e.,

$$g_{\rm str} \to 0; \qquad r_0 \to 0; \qquad \mu = \frac{r_0^2}{g_{\rm str}^2 l_s^2} \to \text{fixed}\,. \tag{2.3}$$

To analyze this limit it is convenient to introduce a new coordinate $r = r_0 \cosh\sigma$ (so that we only focus on the region of the spacetime exterior to the horizon), obtaining

$$
\begin{aligned}
ds^2_{\text{str}} &= -\tanh^2\sigma\, dt^2 + N l_s^2\, d\sigma^2 + N l_s^2\, d\Omega_3^2 + dy_5^2 \\
e^{2\Phi} &= \frac{N}{\mu \cosh^2\sigma}
\end{aligned}
\tag{2.4}
$$

Thus, as is clear from the metric in 2.4, the spacetime is the direct product of the two-dimensional dilaton black hole [10], a three-sphere of constant radius proportional to $\sqrt{N\alpha'}$ and five dimensional flat space. The geometry is perfectly smooth, curvatures being small everywhere (when the number of NS5-branes N is taken to be large) and simultaneously string perturbation theory is also good (so long as $\mu \gg N \gg 1$, since the dilaton is bounded from above by its value at the horizon, $\frac{N}{\mu}$). The above geometry is supposed to be holographically dual to LST at finite temperature [1, 5, 12]. It is also closely related to the description of double scaled LST (DSLST) described in [8, 9].

Continuing to euclidean time, we obtain the the two dimensional cigar, which asymptotes to the linear dilaton vacuum times a euclidean timelike circle. This along with the rest of the spacetime can be described by an exact conformal field theory, with target space

$$
H_3^+/\,\mathrm{U}(1) \times \mathrm{SU}(2)_N \times \mathbb{R}^5\,, \tag{2.5}
$$

where $H_3^+ = \mathrm{SL}(2,C)_N/\,\mathrm{SU}(2)_N$, is the euclidean cigar. Knowing the explicit solution to the metric we can read off as usual the temperature of the black hole by demanding the absence of a conical singularity at the tip ($\sigma = 0$) in euclidean time. This gives

$$
\beta_H = \frac{1}{T_H} = 2\pi\sqrt{N\alpha'}\,. \tag{2.6}
$$

The Hawking temperature of the black hole is thus calculated to be independent of the energy, implying degeneracy of the thermal ensemble. Note that this temperature is exactly the Hagedorn temperature of the LST. The fact that one can tune the energy and the temperature of the system independently indicates the proportionality of the entropy to the energy, implying that the free energy vanishes. The equation of state is the Hagedorn form

$$
S = \beta_H E \tag{2.7}
$$

which leads to an exponentially growing density of states, $\rho(E) \sim e^{\beta_H E}$. In order to understand the thermal ensemble one has to study quantum corrections to the above. Calculation of the one-loop free energy would tell us whether or not the thermal ensemble is stable. This calculation was carried out in a recent work of Kutasov and Sahakyan [16], which we review in the next section.

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Since the background could be understood as a CFT, we can ask how is it that in the CFT we see that the equation of state is as given in 2.7. It was argued in [16] that world-sheet supersymmetry of the CFT is sufficient to guarantee vanishing of the tree-level free energy.[2]

2.2 One loop analysis

We would like to estimate the one-loop free energy of the gravitational background (2.4). One way to go about this would be to do a one loop analog of the Gibbons-Hawking [25] calculation of evaluating the euclidean action about the solution. But this is fraught with the usual problems of regulating the divergences in supergravity. Nonetheless, we are aware of the fact that string theory provides a good regulator mechanism and one would hope that the string partition function evaluated in the background 2.4 would contain valuable information about the thermal ensemble.

Loop/stringy corrections to the Hagedorn density of states of LST were studied in [14, 15, 16]. The basic argument was that finite energy corrections to the Hagedorn density of states, $\rho(E) \sim e^{\beta_H E}$, would be of the form

$$\rho(E) \sim E^{\alpha} e^{\beta_H E} \left(1 + \mathcal{O}\left(\frac{1}{E}\right)\right). \tag{2.8}$$

The primary question is "What is the sign of α?" A negative value of α would indicate that the thermal ensemble is unstable (since the specific heat would be negative). Indeed the explicit calculation of [16] indicated that α was negative; we shall present a brief review of the argument below.

The starting point in [16] was that despite the background 2.4 having a non-trivial dilaton profile, for purposes of computing the torus partition function all one needs to do is to replace the cigar by a cylinder of finite size, because the torus partition function is independent of the string coupling. The torus partition function being extensive, would be proportional to the volume of the cigar and also to the volume of the NS5-brane. The volume of the cigar is divergent, but can be regulated by introducing an ultra-violet cut-off $\phi \leq \phi_{UV}$ (since this divergence is plainly associated with the semi-infinite linear dilaton CHS tube). On the other hand presence of a regular horizon of the two-dimensional black hole (or in euclidean signature, the tip of the cigar) protects the CFT from entering the strong coupling region and also provides an IR cut-off. So forgetting about the dilaton profile we can replace the cigar by a cylinder of length L_ϕ, and radius β_H, where

$$L_\phi = \phi_{UV} - \phi(\sigma = 0) = \phi_{UV} - \frac{1}{2}\log\frac{\mu}{N} = -\frac{1}{2}\log E + \text{const.} \tag{2.9}$$

[2] It is not sufficient in linear dilatonic backgrounds to claim that the tree level partition function vanishes because of the infinite volume associated with the $\mathrm{SL}(2, C)$ conformal Killing group of the sphere, for the volume of the cigar has a similar divergence, see [16] for details.

So one is instructed therefore to compute the free string theory partition function on

$$\mathbb{R}_\phi \times \mathbf{S}^1 \times \mathbb{R}^5 \times \mathrm{SU}(2)_N \tag{2.10}$$

In the large N limit one could replace the three-sphere by $\mathbb{R}^3$, since the radius of the $\mathbf{S}^3$ scales like $\sqrt{N}$. So apart from a factor of L_ϕ which arises because of the extensivity of the partition function, the rest of the calculation amounts to computing free string theory partition function on $\mathbb{R}^9 \times \mathbf{S}^1$, which is basically the same computation as in [24]. The basic result obtained in [16] is that

$$-\beta F = Z_{\text{torus}} = \frac{\beta V_5 L_\phi}{4}\gamma\,, \tag{2.11}$$

where, γ is a positive definite number that can be found in [16]; it is basically the free string partition function evaluated at the Hagedorn temperature of the LST 2.6. Assuming that the density of states is given as in 2.8 we can compute the free energy. It is of the same form as in 2.11, giving,

$$\alpha + 1 = -\frac{\beta V_5}{4}\gamma\,, \tag{2.12}$$

where we have made use of the fact that $L_\phi \sim -\log E$. For an explicit expression the reader is referred to the original paper [16].

3. A question of interpretation

The string theory computation shows that the thermal ensemble has an instability at one loop. Kutasov and Sahakyan [16], interpreted this to be indicative of a stringy mode becoming tachyonic at one loop. In the lorentzian picture of the black hole, the thermal nature of the system appears to be a characteristic feature of the horizon. So a thermodynamic instability in the dual field theory could correspond to a deformation of the black hole horizon. This can be realized in the CFT if there were to exist a state which was potentially unstable to quantum corrections. One would in addition require that its wavefunction be supported near the tip of the cigar, for it to correspond in the geometric language to a deformation of the horizon. To summarize, one is looking for a state in the CFT which is massless at tree level, potentially becoming tachyonic when loop corrections are incorporated and localized near the tip of the cigar.

A state that satisfies the above criteria was found in the analysis of DSLST [8, 9]. It was argued that the two-point functions of operators $\mathcal{O}_m$, given in the $(-1,-1)$ picture as,

$$\mathcal{O}_m = e^{-\varphi-\bar{\varphi}} V_{j;m,m} e^{i\vec{p}.\vec{x}}\,, \tag{3.1}$$

where, $\varphi, \bar{\varphi}$ are the bosonized ghosts, $\vec{p}$ is the spatial momentum along the longitudinal directions of the five-brane and $V_{j;m,m}$ is an observable in the $\mathrm{SL}(2)/\mathrm{U}(1)$

coset CFT of the cigar; have poles which correspond to light states. In particular there exists a massless state which has unit winding number along the euclidean time direction. It was conjectured in [16] that in the absence of spacetime supersymmetry this state is likely to acquire a tachyonic mass term from loop corrections and would therefore be a likely candidate for causing the instability.

The existence of a stringy instability in a smooth geometry wherein supergravity is always valid, is extremely surprising. We would like to claim that this instability is in fact intrinsic to supergravity. To motivate this claim let us first of all understand how one can see the instability in supergravity. We have already mentioned that calculation of supergravity one-loop partition function is rendered iffy by the usual problems of having to regulate divergences. But since string theory provides a natural regulator for supergravity, we shall view the zero mode contribution of the string theory partition function as the regulated supergravity partition function.

To check that the zero mode part of the string theory partition function has the same pathology, one starts from the sigma model with a non-trivial dilaton and separates out the zero mode piece of the dilaton from the fluctuation part and does the path integral over the zero mode. (this manipulation is standard in the context of two-dimensional Liouville theory, cf., [26, 27]). This allows one to isolate the zero mode contribution in an explicit manner and as we shall demonstrate reduce the rest of the problem to a familiar system.

Without loss of generality one can restrict oneself to the cigar part of the geometry. What we have here is a Liouville field coupled to a single compact scalar;

$$\begin{aligned} Z_{\text{torus}} &= \int [D\phi][DX] e^{-S} \\ S &= \frac{1}{4\pi} \int d^2\sigma \sqrt{\hat{g}} \left(\partial_\mu X \partial^\mu X + \partial_\mu \phi \partial^\mu \phi - 4\hat{R}\phi + 4\Delta e^{-2\phi} \right). \end{aligned} \tag{3.2}$$

Δ here is a bare cosmological term and the hatted quantities denote background values. Splitting the ϕ integral into a zero mode and a fluctuation part; $\phi = \phi_0 + \delta\phi$, and integrating out ϕ_0 one obtains [27],

$$Z_{\text{torus}} \sim \left(\frac{\Delta}{\pi} \right)^s \Gamma(-s) \int [D\tilde{X}][D\tilde{\phi}]\, e^{-S_0} \left(\int d^2\sigma \sqrt{\hat{g}} e^{-2\tilde{\phi}} \right)^s , \tag{3.3}$$

where

$$S_0 = \int d^2\sigma \, \sqrt{\hat{g}} \left(\partial_\mu \tilde{X} \partial^\mu \tilde{X} + \partial_\mu \tilde{\phi} \partial^\mu \tilde{\phi} \right) \tag{3.4}$$

is the free action of corresponding to a compact scalar and a non-compact scalar. The quantity s is basically related to the Euler characteristic of the world-sheet and also has contributions from the vertex operator insertions. For the particular case of the torus with no insertions one has $s = 0$, implying

$$Z_{\text{torus}} = -\frac{1}{2} \log E \int_{\mathcal{F}} d^2\tau \int D[\tilde{X}][D\tilde{\phi}] e^{-S_0} . \tag{3.5}$$

We obtained a factor proportional to $-\frac{1}{2}\log E$ from the zero mode part (it is related to a properly regulated version of Δ^s in 3.3). What we have shown here is that the zero mode part of the partition function gives precisely the logarithmic dependence on energy with the right sign; the rest of the partition function is readily seen to be the free string theory contribution which has no pathologies for temperatures of order T_H. It is therefore tempting to argue for a supergravity origin of the instability seen in the thermal ensemble.

4. Supergravity instability

A well known example of classical instabilities in supergravity is the Gregory-Laflamme (GL) instability [18, 19]. The basic argument for a GL instability of a black p-brane is related to the fact that a black brane is entropically less favored when compared with a configuration comprising of an array of black holes, with the same amount of mass and charge per unit volume. We might ask whether such an instability is likely to occur in the case of the LST background 2.4. At first sight this appears unlikely. The GL instability was shown to exist only for branes that are far from extremal. In the original work [19] the numerical approximations were argued to be unreliable close to extremality. In the present case we are starting with non-extremal black 5-branes (we are in fact in the same set up as in [19], since they are also consider NS-NS form fields), but the decoupling limit 2.3 instructs us to take the non-extremality parameter r_0 to zero. Thus naively it would appear that one is beyond the regime of the GL instability that has been investigated and one might be forced to look elsewhere for the origins of the instability.

However, in a recent work by Reall [20], it was argued that the GL instability persists for five-branes even in the near-extremal region (the result is also attributed to [28]). So one would have to be careful in claiming that the spacetime geometry 2.4, which is relevant for the decoupled LST, is GL stable. In fact we shall argue that in the limit $r_0 \to 0$, (note that since we also take $g_{\rm str} \to 0$ we do not reach the extremal NS5-brane which is stable) the GL instability does survive and is responsible for an unstable thermal ensemble. This would in line with the philosophy advocated in [21, 22]. The argument made in these works was that a black brane would be classically stable iff it were thermodynamically stable. In the present case given that we know of a thermodynamic instability in the dual field theory, we would like to associate it with a classical GL instability of the supergravity background.

To make precise what exactly we mean by the geometry 2.4, to be GL unstable, let us briefly recall the analysis of [19]. One starts with a classical black brane background, with a metric $g^{(0)}_{\mu\nu}$, dilaton $\phi^{(0)}$, and a p-form field strength $F_{(p)}$. The black brane solutions are typically of the form $\mathbb{R} \times \mathbb{R}^{8-p} \times \mathbf{S}^p$. Fluctuations about the classical background are considered, in particular one looks at fluctuations that carry some momentum along the longitudinal directions i.e.,

$$\begin{aligned} g_{\mu\nu}(r,t,x) &= g^{(0)}_{\mu\nu}(r) \; + \; e^{i\omega t + ik.x} h_{\mu\nu}(r) \\ \phi(r,t,x) &= \phi^{(0)}(r) \; + \; e^{i\omega t + ik.x} f(r) \end{aligned} \tag{4.1}$$

The instability is typically expected to occur in the s-wave sector of the sphere in question, for higher angular momenta modes would correspond to heavier particles of the KK spectrum on the sphere. Also fluctuations of the p-form field strength are set to zero (see [19] for consistency of this ansatz at first order in perturbations). What the GL instability means is that there are solutions to the linearized equations about the black brane background, wherein the fluctuations are perfectly regular both at the horizon and in the asymptotic region, but have an imaginary frequency i.e., the fluctuations grow exponentially in time as opposed to being oscillatory. Of course, this behavior is not present at all length scales. In fact for small wavelengths there is no instability and the only regular solutions are those that are oscillatory in time. But above a critical wavelength corresponding to a critical longitudinal momentum k_* (where $k^2 = k_i.k^i$), there is an instability i.e., the fluctuation equations admit regular solutions that are exponentially growing in time. At the critical momentum k_* it is argued in [20] that there ought to be a time independent solution to the fluctuation equations, i.e. a zero frequency mode, which we shall refer to as the *threshold unstable mode*, borrowing Reall's terminology.

The analysis of [20] shows that when the fluctuation equations admit a constant time solution, the corresponding euclidean action is negative definite. This indicates a thermodynamic instability, for one can view the euclidean action evaluated about the background to be the thermal partition function *a la* Gibbons-Hawking [25]. In particular the fact that the NS5-branes are GL unstable all the way down to extremality implies the existence of a threshold unstable mode for NS5-branes. We wish to claim that this threshold unstable mode survives the decoupling limit and is responsible for the instability of the thermal ensemble. The conjecture therefore can be stated as

In the decoupling limit of non-extremal NS5-branes, the threshold unstable mode [20] survives. This mode has zero frequency, but it potentially can acquire a tachyonic mass term at one-loop. It is precisely this mode that is responsible for the thermodynamic instability seen in the partition function of [16].

At the present juncture we have no explicit proof for the existence of a threshold unstable mode. We hope to demonstrate the existence of such in the near future.

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